



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

The examination of these very important and interesting particulars has, I observe, drawn me into a prolixity, which I fear may prove tedious to your Lordship: but should I have removed all doubts, and brought convincing proofs of the absurdity of fearing a second infection; should I have shewn inoculation to be a necessary practice, and that the contagious distemper may be communicated more ways than one; I hope your Lordship will excuse the length of this letter. I shall only add my earnest wishes, that the legislature may, by effectual means, prevent the importation of distempered cattle and hides into these kingdoms; the only means of naturalizing and perpetuating a dreadful distemper, now, thank God! much decreased among us.

I am, with the greatest respect,

My Lord,

Your Lordship's

Most humble and most obedient Servant,

Huntingdon,
26 Nov. 1757.

Daniel Peter Layard.

LXX. *Trigonometry abridged.* By the Rev.
Patrick Murdoch, A. M. F. R. S.

Read Feb. 2, 1758. **T**HE cases in trigonometry, that can properly be called different from one another are no more than *four*; which may be resolved by *three* general rules or theorems, expressed

Fig. 1.

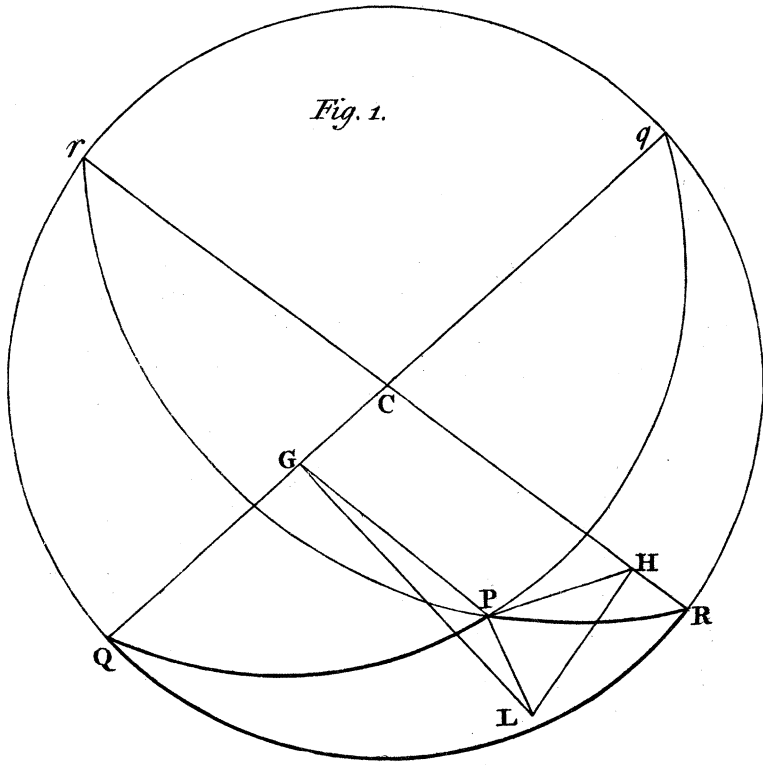
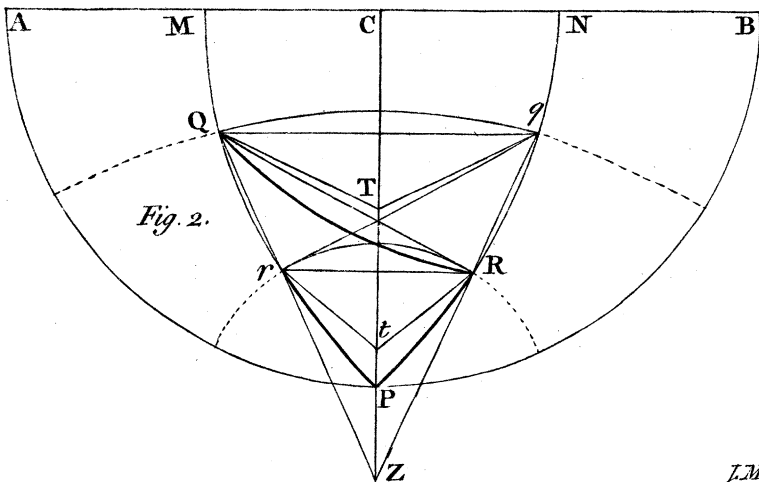


Fig. 2.



in the sines of arcs only; using the supplemental triangle as there is occasion.

C A S E I.

When of three given parts two stand opposite to each other, and the third stands opposite to the part required.

T H E O R E M I.

The sines of the sides are proportional to the sines of angles opposite to them.

D E M O N S T R A T I O N.

Let QR (TAB. XX. Fig. 1.) be the base of a spherical triangle; its sides PQ , PR , whose planes cut that of the base in the diameters QCq , RCr . And if, from the angle P , the line PL is perpendicular to the plane of the base, meeting it in L , all planes drawn through PL will be perpendicular to the same, by 18. *el.* 11. Let two such planes be perpendicular likewise to the semicircles of the sides, cutting them in the straight lines PG , PH ; and the plane of the base in the lines LG , LH .

Then the plane of the triangle PGL being perpendicular to the two planes, whose intersection is $QGCq$, the angles PGQ , LGQ will be right angles, by 19. *el.* 11. PG likewise subtends a right angle PLG , and the angle PGL measures the inclination of the semicircle QPq to the plane of the base (*def.* 6. *el.* 11.) that is (by 16 *el.* 3. and 10 *el.* 11.) it is equal to the spherical angle PQR : whence PG is to PL as the radius to the sine of PQR . The same way PL is to PH as the sine of PRQ is to

Z z z z

the

the radius : and therefore, *ex æquo*, PG the sine of the side PQ is to PH the sine of PR, as the sine of PRQ is to the sine of PQR.

CASES II. and III.

When the three parts are of the same name:

And,

When two given parts include between them a given part of a different name, the part required standing opposite to this middle part.

THEOREM II.

Let S and s be the sines of two sides of a spherical triangle, d the sine of half the difference of the same sides, a the sine of half the included angle, b the sine of half the base; and writing unity for the radius, we have $Ss a^2 + d^2 - b^2 = 0$; in which a or b may be made the unknown quantity, as the case requires.

DEMONSTRATION.

Let PQR (Fig. 2.) be a spherical triangle, whose sides are PQ PR, the angle included QPR, the base QR, PC the semiaxis of the sphere, in which the planes of the sides intersect.

To the pole P, draw the great circle AB, cutting the sides (produced, if needful) in M and N; and thro' Q and R, the lesser circles Qq, rR, cutting off the arcs Qr, qR equal to the difference of the sides; join MN, Qq, rR, QR, qr.

Then the planes of the circles described being parallel (*Theod. sphæric. 2. 2.*), and the axis PC perpendicular to them (10. 1. of the same), their intersections

tions with the planes of the sides, as QT , and Rt , will make right angles with PC ; that is, QT and Rt are the sines (S, s) of the sides PQ PR , and MC NC are whole sines. Now the isosceles triangles MCN , QTq , rtR , being manifestly similar; as also MN , the subtense of the arc which measures the angle QPR , being equal to $(2a)$ twice the sine of half that angle; we shall have $MN : MC :: Qq : QT :: rR : Rt$; or, in the notation of the theorem, $Qq = 2Sa$, $rR = 2sa$. And further, the chords Qr qR being equal, and equally distant from the center of the sphere, as also equally inclined to the axis PC , will, if produced, meet the axis produced, in one point Z . Whence the points Q , q , R , r , are in one plane (2. *el.* 11.), and in the circumference in which that plane cuts the surface of the sphere: the quadrilateral $QqRr$ is also a segment of the isosceles triangle ZQq , cut off by a line parallel to its base, making the diagonals QR , qr , equal. And therefore, by a known property of the circle, $Qq \times rR + \overline{qR}^2 = \overline{QR}^2$; which, substituting for Qq and Rr the values found above, $2d$ for Qr , $2b$ for QR , and taking the fourth part of the whole, becomes $Ssa^2 + d^2 = b^2$, the proposition that was to be demonstrated.

Note 1. If this, or the preceding, is applied to a plane triangle, the sines of the sides become the sides themselves; the triangle being conceived to lie in the surface of a sphere greater than any that can be assigned.

Note 2. If the two sides are equal, d vanishing, the operation is shorter: as it likewise is when one or both sides are quadrants.

Note

Note 3. By comparing this proposition with that of the Lord Neper †, which makes the 39th of Keill's Trigonometry, it appears, that if AC, AM, are two arcs, then $\text{fin. } \frac{AC + AM}{2} \times \text{fin. } \frac{AC - AM}{2} = (\overline{b + d} \times \overline{b - d} =) \text{fin. } \frac{1}{2} AC + \text{fin. } \frac{1}{2} AM \times \text{fin. } \frac{1}{2} AC - \text{fin. } \frac{1}{2} AM$. And in the solution of Case II. the first of these products will be the most readily computed.

C A S E IV.

When the part required stands opposite to a part, which is likewise unknown : Having from the data of Case I. found a fourth part, let the fines of the given sides be S, s; those of the given angles Σ, σ ; and the fines of half the unknown parts a and b; and we shall have, as before, $Ss a^2 + d^2 - b^2 = 0$; and if the equation of the supplements be $\Sigma \sigma \alpha^2 + \delta^2 - \beta^2 = 0$; then, because $\alpha^2 = 1 - b^2 = 1 - \overline{Ss a^2 + d^2}$, and $\beta^2 = 1 - a^2$, substituting these values in the second equation, we get

T H E O R E M III.

$$\frac{1 - \Sigma \sigma \times \overline{1 - d^2 - \delta^2}}{1 - Ss \Sigma \sigma} = a^2; \text{ in words thus:}$$

Multiply the product of the fines of the two known angles by the square of the cosine of half the difference of the sides: add the square of the sine of half the difference of the angles; and divide the complement of this

† See Logarith. Canon. defer. Edinb. 1614. p. 48.

sum to unity, by the like complement of the product of the four sines of the sides and angles; and the square root of the quotient shall be the sine of half the unknown angle.

If we work by logarithms, the operation will not be very troublesome; but the rule needs not be used, unless when a table of the trigonometrical analogies is wanting. To supply which, the foregoing theorems will be found sufficient, and of ready use; being either committed to memory, or noted down on the blank leaf of the trigonometrical tables.

Note, The schemes may be better, raised in card-paper, or with bent wires and threads.

LXXI. *An Account of Two extraordinary Cases of Gall-Stones.* By James Johnstone, M. D. of Kidderminster. Communicated by the Rev. Charles Lyttelton, L. L. D. Dean of Exeter.

To the Rev. Dr. Lyttelton, Dean of Exeter.

Rev. Sir,

Read Feb. 9,
1758.

According to promise I send you a short account of the two extraordinary cases we talked of, the last time I had the pleasure of seeing you at Kidderminster.

The truth of the first narrated case you are already a sufficient judge of; and if it is at all necessary to ascertain